DESIGNING MATCHED WAVELETS FOR R PEAK DETECTION IN ECG SIGNAL

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Abstract-**A novel method is used to design wavelets matched to the ECG signal. The designed matched wavelet is then used for detecting the R peaks of ECG signal through a new detection algorithm. No pre-processing or filtering is applied; the signals are directly analyzed using the proposed R peak detection algorithm. The algorithm is evaluated on real data from MIT-BIH Arrhythmia database. Experimental results illustrate that the proposed method is faster than the previous methods while having similar accuracy.**

***Keywords* - Wavelet transform, matched wavelet, ECG signal, QRS complex, R peak detection.**

I. INTRODUCTION

Detection of R peak is the first step in processing an ECG signal as it is usually used as a reference within the cardiac cyclic and for beat alignment. So far, a vast number of algorithms have been proposed for the detection of the QRS complex which can be categorized as: 1) syntactic, 2) non-syntactic, and 3) hybrid [1], [2].

The syntactic algorithms are time consuming and are not appropriate for online applications. In the non-syntactic approach, the signal is first filtered by a band pass filter or a matched filter in order to suppress the T and P waves and the noise. The output of the filter is then passed through a nonlinear transformer to enhance the QRS complex. But different subjects and even different beats of the same subject have different QRS frequency bands and also the frequency bands of QRS and noise may overlap.

Wavelet transform is a powerful tool for analyzing non-stationary signals, such as ECG. It provides a time-frequency analysis of the signals by decomposing them into orthogonal frequency channels that have the same bandwidth on a logarithmic scale. These 'building blocks' are well localized both in time and frequency .This feature of the wavelet transform is very effective for feature extraction and classification [3], [4]. Using the wavelet transform, the variations in frequency components of an ECG signal can be followed. Thus the accuracy and reliability of the ECG feature extraction improves significantly.

Wavelet-based methods for R peak detection have mainly used quadratic spline and Daubechies wavelets which have lead to excellent results and robustness against noise and baseline drift. Many different wavelet bases with different characteristics have been designed and can be used for ECG processing [4]-[7]. In addition, it is possible to directly design the appropriate (matched) wavelet for a particular application. Matched wavelets can be designed such that the error between the original signal and some finite resolution wavelet representation of it is minimized [4]-[6].

This paper is organized as follows. In Section II, we present an introduction to the wavelet transform and then explain the design of matched wavelets. In this section, we also present a new fast and reliable approach for R peak detection which is compatible with different wavelet bases. In Section III, through experimental studies, we show the power of the new designed matched wavelet and also the new approach for R peak detection using the standard, annotated MIT-BIH Arrhythmia ECG database. Section IV is our concluding remarks.

II. METHODOLOGY

The wavelet transform (WT) of a signal *f(x)* is a decomposition of the signal as a combination of a set of basis functions [1]. The basis functions are the dilated and translated versions of a single prototype wavelet function *Ψ(t)*. So $W_s f(x)$, the WT of the signal, is defined as:

$$W_s f(x)=f(x)*\psi_s(x)= s^{-1}\int_{-\infty}^{+\infty} f(t)\psi(\frac{x-t}{s})dt \quad (1)$$

The greater the scale factor *s*, the wider the basis function which corresponds to the lower frequency components of the signal and vice versa. If we choose the scaling factor *s* such that $s=2^j$ ($j\in Z$), then the WT is called dyadic WT. Regarding Mallat algorithm, the dyadic WT of a discrete-time signal (DWT) is equivalent to an octave filter bank and thus can be calculated as follows:

$$S_{2^j} f(n)=\sum_{k\in Z} h_k S_{2^{j-1}} f(n-2^{j-1}k) \quad (2)$$

$$W_{2^j} f(n)=\sum_{k\in Z} g_k S_{2^{j-1}} f(n-2^{j-1}k) \quad (3)$$

S_{2^j} is a smoothing operator where S_{2^0} is equal with the digital signal to be analyzed and h_k, g_k are the coefficients of the low pass and high pass FIR filters H(w) and G(w),respectively. $S_{2^j} f(n)$ can be interpreted as the approximation of the original signal at the j^{th} scale whereas $W_{2^j} f(n)$ demonstrates the details of the signal at that scale.

The high pass and low pass filters H (w) and G (w) depend on the selected wavelet prototype function. Let $[h_0, h_1, ..., h_{N-1}]$ and $[g_0, g_1, ..., g_{N-1}]$ be the impulse response of the low pass and high pass filters, respectively. The following conditions ensure orthogonality of the transform so that no information is lost in the decomposition process and the filter bank is said to enjoy a perfect reconstruction (PR) property [8].

$$\sum_{n=0}^{N-1-2l} h_n h_{n+2l} = \begin{cases} 1 & ,l=0 \\ 0 & ,0<l<N/2 \end{cases} \quad (4a)$$

$$g_n = (-1)^{n+1} h_{N-n-1} \quad , n = 0,1,..., N-1 \tag{4b}$$

2) Designing Matched Wavelet

The objective function of the methods proposed in [5]-[7] is the squared approximation error between the signal of interest and its approximation at some finite prescribed scale. Since the approximation error is not generally an explicit function of the design parameters, an approximation of this objective function is required so that design problem is converted to a conventional optimization problem [5] - [7].

The matching algorithm proposed in [5] is sub-optimal in as it is performed on the spectrum magnitude and phase independent of one another. The algorithm proposed in [6] has a more accurate approximation of the squared error than the algorithm presented in [7], so this algorithm is chosen among the proposed algorithms in [5]-[7] for designing wavelets matched to the ECG signal.

For the given signal *f(t),* the prescribed scale (resolution) *J,* and the dilation factor *M*, the algorithm seeks for the optimal M-band wavelet multi-resolution that represents *f(t)* at scale *J*. The optimality is measured with respect to minimization of frequency norm of the approximation error. In this work, 2-band wavelet analysis and synthesis is implemented (M=2).

M-band compactly supported orthonormal wavelet bases can be uniquely characterized by a unitary sequence known as the unitary scaling vector, h_0. For the N-length scaling vector, the following linear and quadratic constraints exist:

$$\sum_{k=0}^{N-1} h_0(k) = \sqrt{2} \tag{5}$$

$$\sum_{k=0}^{N-1} h_0(k) h_0(k+Ml) = \delta(l) \tag{6}$$

Given a length N and the scaling vector h_0 , there exists a unique, compactly supported scaling function $\varphi_0(t) \in L^2(R)$ that satisfies the following recursion:

$$\varphi_0(t) = \sqrt{2} \sum_{k-0}^{N-1} h_0(k) \varphi_0(2t-k) \tag{7}$$

Due to multi-resolution theorem, any function *f(t)* $\in L^2(R)$ can be uniquely represented in the following form:

$$f(t) = \sum_k c_{j_0}(k) 2^{j_0/2} \varphi_0(2^{j_0} t - k) + \sum_k \sum_{j=j_0}^{\infty} d_j(k) \psi_{j,k}(t) \tag{8}$$

Where

$$\psi_{j,k} = 2^{j/2} \psi(2^j t - k) \tag{9}$$

Therefore, all functions *f (t)* can be practically approximated at scale J as

$$f(t) \approx \sum_k c_{J,k} 2^{J/2} \varphi_0(2^J t - k) \tag{10}$$

so the approximation error can be defined as

$$Qf(t) = f(t) - \sum_k c_{J,k} 2^{J/2} \varphi_0(2^J t - k) \tag{11}$$

Using the parameterization method proposed in [6], the scaling vector h_0 of length N=MK (M=2), can be parameterized by K-1 unit vectors v_i. Each unit vector itself is parameterized by M-1 angle parameters $\theta_{i,k}$, i=0,… , K-1; k=0, 1,… , M-2. Following the mathematical steps and lemmas in [6], the design of optimal multi-resolution analysis, with the L_p error norm, is converted to the following optimization problem:

$$\min_\theta \left\{ \int_\Omega dw \left(\left| 1 - \left| \hat{\varphi}_0\left(\frac{w}{2^J}\right) \right|^2 \right|^p + \left| \hat{\varphi}_0^*\left(\frac{w}{2^J}\right) \right|^p \sum_{k \neq 0} \left| \hat{\varphi}_0\left(\frac{w + 2\pi 2^J k}{2^J}\right) \right|^p \right) \right\} \tag{12}$$

where $\hat{\varphi}_0$ is the Fourier transform of $\varphi_0(t)$.

Once the optimization problem stated in (12) is solved, the scaling vector, the scaling function and thus the wavelet basis is uniquely obtained.

3) Designing Wavelet Matched to ECG Signal

In order to preserve compatibility with the previous related works, MIT-BIH Arrhythmia, which is a standard annotated ECG database, is selected as the data set both for training and testing the proposed algorithm. This database contains 48 real ECG records which are 30 minutes long and have been sampled at 360 Hz.

A typical cardiac cycle is selected from the first record of this database (record#100). To follow the recommendations of the AAMI (Association for the Advancement of Medical Instrument), the first 5 minute of this record is removed and the first cardiac cycle appearing after this duration is selected. This interval is 180 points long and its R peak is located at the centre of the interval. This interval is selected as the signal of interest for matching the wavelet.

The length of the wavelet is set to N=8, the average of the lengths of the wavelets used in previous works. The target function is the approximation error at the second scale (J=2). Following the steps discussed in the previous sub-section, the wavelet matched to this cardiac cycle is designed. The impulse response and frequency response of the corresponding low-pass filter, h, are shown in Fig. 1.

4) Wavelet-Based R Peak Detection Algorithm

The algorithm proposed in [1], [2] is the basis of R peak detection in this work. The algorithm is applied directly to the digitized signal without any filtering or pre-processing. This algorithm contains three main parts: 1) windowing the ECG signals to analyze 720 data points at a time, 2) computing the DWT of each interval at the scales 2^2 and 2^1 , and 3) analyzing the wavelet coefficients for R peak detection.

According to signal detection theory, if the system response of the detection filter matches the signal embedded in noise, the signal-to-noise ratio (SNR) is maximized, and a sharp and high peak will be produced at the output showing the maximum correlation [12]. So using the matched wavelets for decomposing the ECG signal generates sharp positive maxima- negative minima at the locations of R peaks at different scales as shown in Fig. 2.

Concerning the power spectra of the ECG signal, noise and artifact and the 3-db bandwidth of the equivalent wavelet analysis filter and in order to decrease the volume of mathematical computations for generating DWT coefficients at different scales, it seemed sufficient to analyze the wavelet

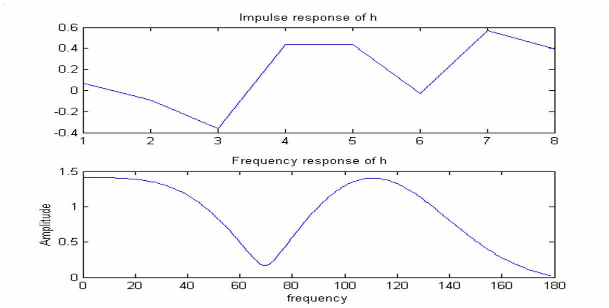

Fig. 1. Impulse response and frequency response of the low-pass filter which corresponds to the matched wavelet.

coefficients only at the scales 2^2 and 2^1 .

The R peak detection algorithm can be divided into the following steps:

Step1: The DWT of the signal is calculated at scales 2^2 and 2^1 .

Step 2: All the maxima in DWT larger than a threshold, which is 2.5 times larger than the rms of DWT coefficients at each interval, at scale 2^2 are detected.

Step 3: All of the maximum-minimum pairs in DWT larger than a threshold at 2^2 scale are found. Similar to step 2, the threshold is updated after each detection.

Step 4: In the neighboring of each maximum-minimum pair detected at scale, a similar pair is detected at scale 2^1.

Step 5: Isolated peaks at scale 2^1 are eliminated.

Step 6: Redundant peaks at scale 2^1 are eliminated.

Step 7: The remaining pairs at scale 2^1 correspond to the R peaks of the signal. The zero crossing points of these pairs are calculated and shifted 2 points to the left to obtain the locations of the R peaks in the ECG signal.

III. Results

The proposed algorithm is evaluated on the standard and manually annotated MIT-BIH Arrhythmia database. No pre-processing of filtering is applied and the signals are directly analyzed using the proposed R peak detection algorithm.

To compare the efficiency of the designed wavelet with the previously used wavelets, the second experiment is performed using quadratic spline wavelet which had led to the best results among those of the previously proposed wavelet-based methods. In order to obtain a meaningful comparison among the different wavelets, the R peak detection algorithm remains unchanged through these experiments.

Table 1 shows the results of the experiments for the MIT-BIH Arrhythmia data-base. “FP” is the number of false positives (incorrectly detected) and “FN” is the number of false negatives (missed) R peaks. The total number of failed detections (FD) is the sum of FP and FN.

IV. Conclusion

This work presented a new method for wavelet-based R peak detection by using matched wavelets. The previous works required analyzing the wavelet coefficients in 4 different scales while the proposed method needs analyzing the coefficients of only 2 scales.

The minimum execution time for the previous methods was about 108 seconds while for the proposed method it was reduced to 26.2 seconds. So, the speed of the algorithm is increased considerably. The results show a considerable increase in the accuracy of the R peak detection algorithm when using matched wavelet instead of Quadratic Spline wavelet, which had led to the best results among the previous wavelet-based algorithms. This is especially true for the number of incorrectly detected R peaks, i.e., using matched wavelet decreases the probability of false detection notably. Future work will use further processing of the wavelet coefficients in order to increase the efficiency and reduce the effect of noise in the ECG signals.

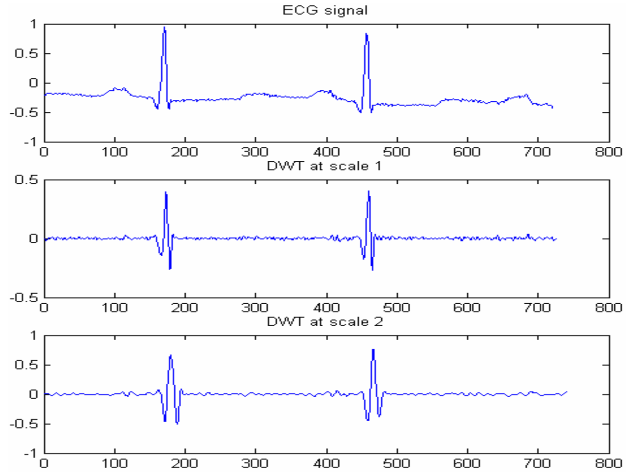

Fig. 2. ECG signal and its DWT at scales 2^1 and 2^2. The sharp maximum peaks at scales 2^1 and 2^2 correspond to R peaks.

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TABLE I
THE RESULTS OF THE R PEAK DETECTION FOR THE MIT/BIH DATABASE. RESULTS #1 CORRESPOND TO THE EXPERIMENT USING MATCHED WAVELET AND RESULTS #2 CORRESPOND TO THE EXPERIMNET USING QUADRATIC SPLINE WAVELET.

Signal	Total beats	FP(#1)	FP(#2)	FN(#1)	FN(#2)	FD (#1)	FD(#2)
100	2273	0	0	0	0	0	0
101	1865	2	1	1	2	3	3
102	2187	0	0	2	3	2	3
103	2084	0	0	4	0	4	0
104	2230	181	137	52	57	233	194
105	2572	24	21	32	26	56	47
106	2027	0	1	101	100	101	101
107	2137	94	651	5	486	99	1137
108	1763	433	525	113	132	546	657
109	2532	4	3	7	184	11	187
111	2124	5	8	4	6	9	14
112	2539	2	0	0	0	2	0
113	1795	0	0	1	0	1	0
114	1879	6	9	7	9	13	18
115	1953	0	0	0	0	0	0
116	2412	2	13	83	48	85	61
117	1535	1	2	1	1	2	3
118	2275	19	4	8	3	27	7
119	1987	0	0	4	445	4	445
121	1863	4	2	8	2	12	4
122	2476	0	0	0	0	0	0
123	1518	0	0	3	2	3	2
124	1619	3	33	5	4	8	37
200	2601	108	118	174	64	282	182
201	1963	9	9	26	66	35	75
202	2136	1	0	140	162	141	162
203	2982	101	124	168	119	269	243
205	2656	4	0	32	33	36	33
207	1862	298	317	56	90	354	407
208	2956	23	19	422	642	445	661
209	3004	11	6	1	18	12	24
210	2647	30	14	38	77	68	91
212	2748	1	2	0	3	1	5
213	3251	5	1	12	175	17	176
214	2208	18	8	16	26	34	34
215	2154	11	9	8	16	19	25
217	2208	11	14	16	42	27	56
219	2154	4	6	2	26	6	32
220	2048	0	0	23	30	23	30
221	2427	3	1	14	24	17	25
222	2484	12	16	44	58	56	74
223	2605	0	0	9	11	9	11
228	2053	363	168	40	21	403	189
230	2256	19	0	2	0	21	0
231	1886	0	0	1	1	1	1
232	1780	793	588	1	5	794	593
233	3079	43	0	57	71	100	71
234	2753	0	0	0	2	0	2
total	116137	2648	2830	1743	3292	4391	6122