

Comparison of Linear and nonlinear Models for Estimating Brain Deformation during Surgery Using Optimization Process

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Abstract

This paper presents finite element computation for brain deformation during craniotomy. The results are used to illustrate the comparison between two mechanical models: linear solid-mechanic model, and non linear finite element model. To this end, we use a test sphere as a model of the brain, tetrahedral finite element mesh, two models that describe the material property of the brain tissue, and function optimization that optimizes the model's parameters by minimizing distance between results deformation and supposed deformation. Both models assume finite deformation of the brain after opening the skull. We compare accuracy of the two models using error of the optimization process.

Keywords

Finite element method, optimization process, brain model.

1. Introduction

Mechanical property of very soft tissue such as brain, liver, and kidney has been studied in recent years. This is because of applications such as surgical robot control system [1], surgical operation planning, and surgeon training systems based on the virtual reality techniques. However, in a common neurosurgical procedure of the brain, it deforms after opening the skull, causing misalignment of the subject to the preoperative images such as magnetic resonance image (MRI) or computed tomography (CT) images [2], [3]. This phenomenon happens because of cerebrospinal fluid (CSF) leakage, dura opening, anaesthetics and osmotic agents, as well as conditions which are different from the normal state. Opening the scalp and CSF leakage cause the gravitational shift of the tissue due to disappearance of tension and pressure forces on the boundary condition of the brain [4], [5]. While the intraoperative imaging is the best way to determine this deformation, intraoperative images suffer from the constraints of the operating room. Thus, spatial resolution and contrast of intra operative images are typically inferior to those of preoperative ones [6]. This problem can be solved by using biomedical models.

To this end, two models have been proposed in recent years, as described next. In 1999 K. Miller [7] suggested a model based on equation of equilibrium that related the covariant differentiation of stress with respect to the deformed configuration to body force per unit mass. In this model brain deformation is supposed to be large, brain tissue is treated as a hyper viscoelastic material and the stress-strain behavior of the brain tissue is non-linear [8], [9]. In 2002, M. Ferrant [10] proposed a mechanical model based on this principle that the sum of the virtual work from the internal strains is equal to the work from the external loads. In this formulation, brain deformation is supposed to be infinitesimal, brain tissue is treated as an elastic material, and the relation between strain and stress is linear [11].

In most practical cases, such models utilize the finite element methods [12] to solve sets of partial differential equations governing the deformation behavior of the tissue. Using these methods, one can define the brain deformation but the brain's parameters are unknown. Previous works used the approximate value of the brain parameters, which we also use in this work. We apply function optimization to optimize these parameters and minimize the distance between the resulting deformation and the supposed deformation.

In this paper we use the above two models of the brain and optimize their parameters to match their resulting deformation with the assumed deformation. We then compare the two models using their resulting errors. In the next section, we explain the models and describe how to use meshing and boundary conditions for solving the problem using finite element methods and how to use function optimization to optimize their parameters. In Section 3 we explain the results of our implementation on a sphere as a model of the brain and compare the methods. Section 4 presents the conclusions of our work.

2. Materials and methods

2.1. Construction of Finite Element Mesh

Within Finite Element Modeling (FEM) framework, the body on which one is working needs to be discretized using finite element mesh. By partitioning the object into small elements, the equations will be solved for every element. Therefore, it will be solved for the whole object. To this end, we used a sphere with a diameter of 22 Cm which is approximately the size of the brain. We also use FEMLAB3.3 to generate 4-noded tetrahedral mesh with Lagrange shape function (Figure 1). This software generates automatic mesh and also by changing its parameters the user can change the mesh size.

2.2. The Computational Biomedical Models

As mentioned before, for determining the deformation of the brain, a model for the brain may be used. Such a model gives some numerical formulations that can describe the behavior of the brain tissue. These formulations can be linear and non-linear. The linear model is simpler to implement [13], [14] and needs less time but a nonlinear model is more complicated. In this section we illustrate two models: one model describes the tissue behavior linearly and the other assumes the brain tissue to be nonlinear.

2.2.1. Linear Solid-Mechanic Model

In this model, the body is assumed to be a linear elastic continuum with no initial stresses or strains. The energy of the body's deformation caused by externally applied forces can be expressed as [10]:

$$E = \frac{1}{2} \int_{\Omega} \sigma^T \varepsilon d\Omega + \int_{\Omega} F^T u d\Omega, \quad (1)$$

Where $F = F(x,y,z)$ is the total force applied to the elastic body, Ω is the elastic body, u is the displacement vector, and ε is the strain vector that can be defined as:

$$\varepsilon = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}, \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}, \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}, \frac{\partial u}{\partial x} + \frac{\partial u}{\partial z} \right)^T = Lu \quad (2)$$

Also, σ is the stress vector and in the case of linear elasticity, with no initial stresses or strains, relates to the strain vector by the linear equation $\sigma = D\varepsilon$ where D is the elasticity matrix describing the material properties [12]. The value of D depends on two material parameters: the Young modules and the Poisson ratios. Volumetric deformation of the brain is founded by solving equation

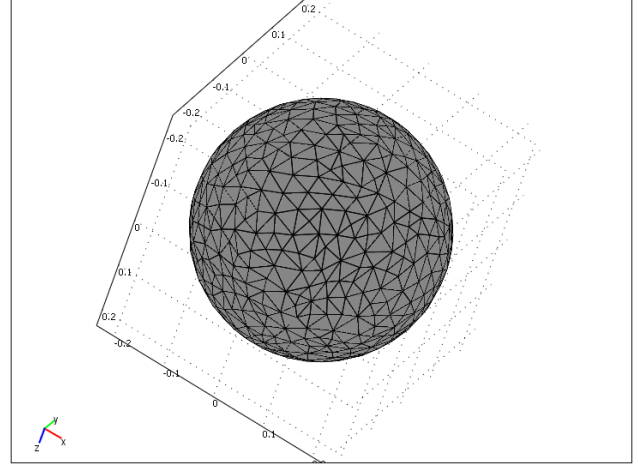


Figure 1. The result of meshing the volume

(1) for the displacement vector u , which minimizes the energy function E . Numerical solution to this equation could be written in a global linear equation:

$$Ku = -F \quad (3)$$

The solution of equation (3) provides us the deformation field that is results from the forces applied to the body. We rely on the study of Ferrant *et al* in [10] and choose our initial coefficients (Young modules = 3 kPa, Poisson ratio = 0.45).

2.2.2. Non-Linear Formulation Model

model the brain is supposed to be a single-phase continuum undergoing large deformations. In this analysis, the stresses and strains are measured with respect to the current configuration. Therefore, using Almansi strain and Cauchy stress, the virtual work principle can be written in the following way [15]:

$$\int_V \tau_{ij} \delta \varepsilon_{ij} dV = \int_V f_i^B \delta u_i dV + \int_S f_i^S \delta u_i dS, \quad (1)$$

Where $\int_V \tau_{ij} \delta \varepsilon_{ij} dV$ is the internal virtual work of strain,

$\int_V f_i^B \delta u_i dV$ is the virtual work of external force that apply

to whole body, and $\int_S f_i^S \delta u_i dS$ is the virtual work of

external force that apply to the surface. As the brain undergoes finite deformation, current volume V and surface S , in the integration of equation (4) is unknown. Therefore, this equation needs to use another equation that describes the mechanical property of the material i.e. appropriate constitutive models. Equation (4) forms a so-called weak formulation of the problem.

An alternative, so-called strong formulation is given by differential equation: [16]

$$\nabla \cdot \tau + \rho F = 0, \quad (2)$$

Where τ denotes Cauchy stress, ρ is a mass density, F is a body force unit mass. Einstein summation convention was used. Differential equation (5) must be supplemented

TABLE I

Table 1. List of material constants for model of brain tissue.

Instantaneous response	Characteristic time t1 = 0.5 (s)	Characteristic time t2=50 (s)
C100= 263 (Pa)	g ₁ =0.450	g ₂ =0.365
C010= 263 (Pa)		
C200= 491 (Pa)		
C020= 491 (Pa)		

by formulae describe the mechanical property of the materials, relating the stress to the deformation of the body.

There exists a variety of methods to solve integral equations (4) and strong formulation (5). Boundary Element Method used the weak form but this method is not suitable for large deformations and nonlinear materials, rather for quasi-static small deformation. Therefore, in this paper we use the strong equation that is appropriate for biomedical engineering application.

As shown by [7], [17] the stress-strain behavior of the brain tissue is nonlinear. This model is suitable for low strain-rates typical for surgical procedures. In this paper we use the model suggested in [7]:

$$W = \int_0^t \left\{ \sum_{i+j=1}^N \left[C_{ij0} \left(1 - \sum_{k=1}^n g_k (1 - e^{-(t-\tau)/\tau_k}) \right) \right] \right\} \times \frac{d}{d\tau} [(J_1 - 3)^i (J_2 - 3)^j] d\tau \quad (3)$$

Where τ_k is characteristic time, g_k is the relaxation coefficient, N is the order of polynomial in strain invariants and J_1 , J_2 , and J_3 are strain invariants:

$$J_1 = \text{Trace}[\mathbf{B}],$$

$$J_2 = \frac{J_1^2 - \text{Trace}[\mathbf{B}^2]}{2J_3}, \quad (4)$$

$$J_3 = \sqrt{\det \mathbf{B}}$$

\mathbf{B} is left Cauchy-Green strain tensor. In this paper we use the stationary form of the equation (6) because we solve the problem for the steady state form of deformation when the deformation of the brain is finished. The initial value of model's parameters are taken from [7] for $n=2$, $N=2$ as summarized in Table I.

2.3. The Optimization Process

However, the parameters of brain in every model are not certain for every people and usually approximated parameters are used. In this paper, we use optimization process to optimize these parameters to achieve the best result in comparison to defined deformation. To this end we choose a cost function that can be determined by the sum of displacement between the defined deformation of some special points and the deformation of those points from results of two models. We use Matlab optimization toolbox for optimization procedure.

The displacement of special points can be defined by surgeon or imaging device such as MRI, CT, or spectroscopic camera. As mentioned before, the resolution of these images in operative room is not good

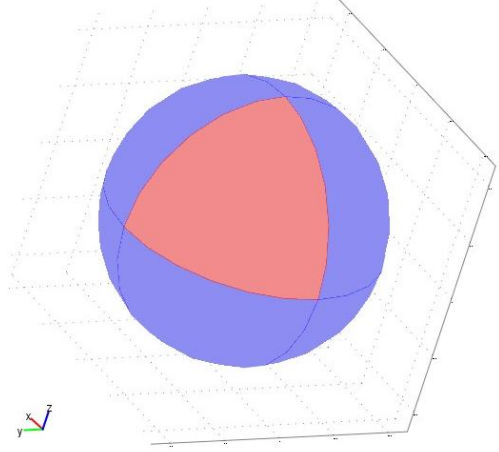


Figure 2. Boundary condition for all models

and because of that it would be better to make images from special part e.g. exposed surface of the brain or some parts that were much important to surgeon like tumor.

In both methods we have some parameters to optimize. In first model, we can't determine the force applied to the exposed surface of brain. This parameter can be defined by optimization process. Two parameters: Young modulus and Poisson's ratio are determined in other paper but they are not certain for every people so, these two parameters will be optimized.

In second model, like first one we can't determine the force applied to the expose surface so, this parameter would be determined by optimization function. Also, the parameters in table (1) that are not certain for every people are used as optimization parameters to minimize the cost function.

By using more defined points the accuracy of our model will be better and the estimated displacement for points inside the brain such as tumor would become more precise.

2.4. Boundary Conditions

For solving partial differential equations we need some boundary condition. As mentioned before, for testing our method we use a sphere as a simple model for the brain and for modeling the craniotomy we assume one section on the sphere to be exposed. So, this section would be free and the rest of them are fixed (Figure 2).

For the first model we have conditions for displacement variable and F (force per unit). The boundary nodes that are not exposed are fixed. We use $F=u$ for boundary conditions of fixed boundary nodes because the elements of the rigidity matrix K in equation (3) that the deformation is supposed to be known need to be set to zero, and the diagonal elements of these rows to one. More detailed can be found in [10].

Also, for the second model boundary condition for F in addition to displacement (u) must be determined. Because this model is nonlinear we can't summarize the

equations to one equation like equation (3). So, we can not determine it and it would be an optimization parameter.

By using these conditions for implementing models the deformation of whole brain can be determined and that can be used to determine some important part like tumor.

3. Results

For modeling the brain, we use a sphere with the diameter of 22 Cm that is approximately the size of the brain. To show the skull opening, we assume that one section of this sphere is exposed and others are fixed. We assume a model with special parameters and define the deformation of some points. Then we change the parameters and use the optimization process to estimate the assumed model's parameters by using displacement of some points. This can show us how much the optimization process can estimate the real parameter of brain by using the displacement of some points. For each model we do this process and compare the accuracy of models using error of optimization function and the error of displacement of some other points that we do not use in optimization process. To implement models, we use the FEMLAB3.3 software which is based on the finite element methods for solving partial differential equations. This software has visualization, meshing, solving the problem, and strong post processing modules.

Figures 3-4 show the first and second model's result. As can be seen both model show the deformation of brain continuously. The first model optimization error is 0.1172 mm, the error for other points that we do not use in optimization process is 0.2731 mm. The second model optimization error is 0.0683 mm, the error for other points that we do not use in optimization process is 0.1915 mm. Therefore, the accuracy of second model is better than the first one but it must be consider that the implementation time of second model is approximately six time more than the first one and in this project the time is one of important factor. This happen because the first model is linear but the second model is nonlinear and more complicated than the first one.

In sum, the second model can estimate the parameters more accurately but it take much more time to implement and this is because of the complication of this model.

4. Conclusion

Mathematical modeling and computer simulation have proved successful in engineering. Computational mechanics has enabled virtual technology in several fields. One of the greatest challenges for mechanical problem is the success of computational mechanics in particular field of biomedical science and biomechanics. In computational science, the most important part in the solution of the problem is the selection of the appropriate physical and mathematical model to be investigated. Model selection is a subjective process, based on analyst's judgment and experience. Nevertheless, model selection is an important step in obtaining valid results. To this end, we choose two linear and nonlinear

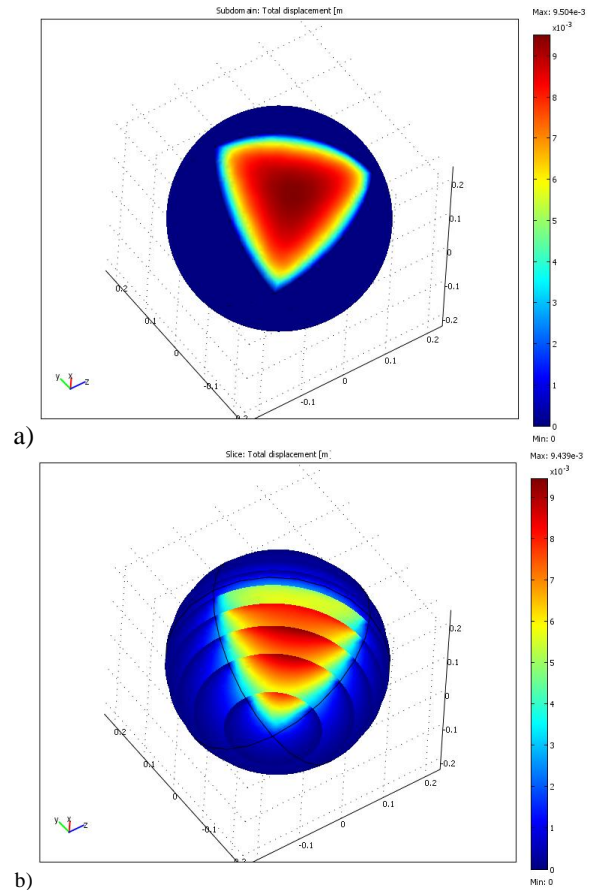


Figure 3. Result of the first model, (a) the total displacement of subdomain and (b) the slice plot of total displacement

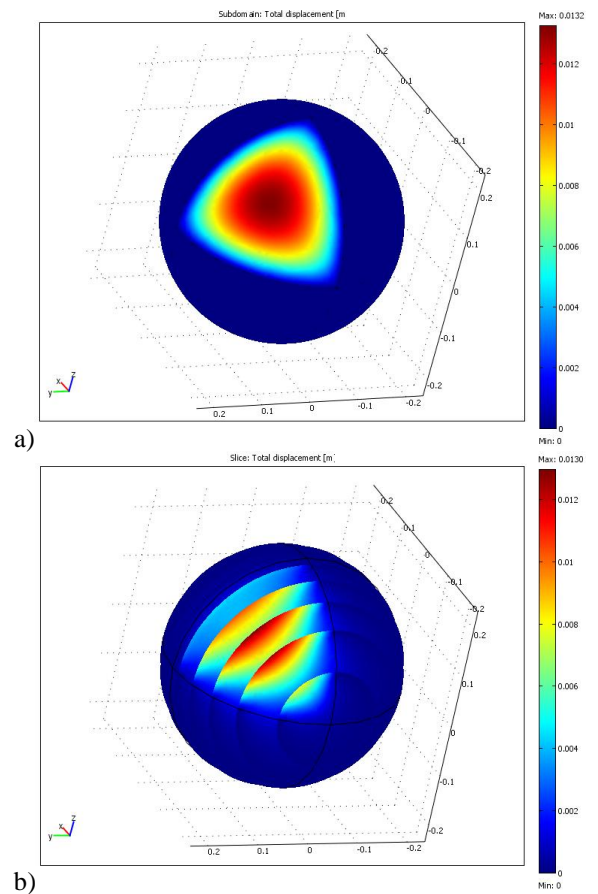


Figure 4. Result of the second model (a) the total displacement of subdomain and (b) the slice plot of total displacement

mechanical models that describe the mechanical property of brain based on finite and large deformation respectively and implement them on a simple sphere. The first model is based on this principle that the sum of virtual work from internal strains is equal to work from external loads and the second model is based on equation of equilibrium that related the covariant differentiation of stress to body force. The accuracy of nonlinear model is better than the linear one but it is too complicate and it's implementation time is much more than the second model. Therefore, depend on the condition of surgery each model have good accuracy and can be used. By using the result of implementing these models on a sphere, we can select the best model for estimating the deformation of the brain.

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