Designing Matched Wavelets with the Maximum Coding Gain Criterion for R Peak Detection in ECG Signal

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Abstract: *Wavelet-based methods have led to the most accurate feature extraction results, especially in the non-stationary signals, among the recent proposed methods. The methods proposed so far have used standard and unmatched wavelet bases such as Daubechies and quadratic spline. In this work, two new matched wavelet bases with the maximum gain criterion are designed and used for analyzing and R peak detection in the ECG signals. A modified wavelet-based R peak detection algorithm is also presented and evaluated using MIT-BIH Arrhythmia database. Experimental results show both excellent performance and high speed of the proposed methods.*

Key Words: Wavelets transform, matched wavelets, coding gain, R peak detection, optimization

1 Introduction

R peak detection is the first step in processing an ECG signal. So far, a vast number of algorithms have been proposed for QRS detection which can be categorized as follows: 1) syntactic, 2) non syntactic and 3) hybrid [1], [2]. The syntactic algorithms are time consuming and are not appropriate for online applications. The non-syntactic methods are applied in the frequency domain, but since ECG is not a stationary signal, these methods are not capable of following the variations in its frequency components. Wavelet transform is a powerful tool for analyzing non-stationary signals, such as ECG, and helps to improve the accuracy and reliability of ECG feature extraction [1]-[3]. Wavelet-based methods have mainly used quadratic spline and Daubechies wavelets which have led to excellent results and robustness against noise and baseline drift. However, many different wavelet bases with different characteristics have been designed and one can choose any of them for processing applications [4]-[7]. What's more, one can directly design the appropriate wavelet that meets the required characteristics.

In our previous work [8], we implemented the wavelet-based R peak detection using Quadratic Spline wavelets. The algorithm was similar to that of [1], but with more flexibility. In our next work, presented in [9], the idea of using matched wavelets for ECG feature extraction was applied for the first time. But the wavelet bases were matched to each interval of the ECG signal which was being analyzed at a time, so a fix wavelet base was not used for analyzing the ECG signals. In this work, two different wavelet bases matched to the ECG signal have been used for decomposing all the ECG signals and detecting their R peaks. Using these wavelets helps increasing the speed of R peak detection significantly while maintaining the quality of the results.

2 Methods and Materials

2.1 An Introduction to Wavelet Transform

Regarding Mallat algorithm, the dyadic WT of a discrete-time signal (DWT) is equivalent to an octave filter bank and thus can be calculated as follows:

$$S_{2^j} f(n) = \sum_{k\in Z} h_k S_{2^{j-1}} f(n - 2^{j-1}k) \qquad (1)$$

$$W_{2^j} f(n) = \sum_{k\in Z} g_k S_{2^{j-1}} f(n - 2^{j-1}k) \qquad (2)$$

S_{2^j} is a smoothing operator where S_{2^0} is equal with the digital signal to be analyzed and h_k, g_k are the coefficients of the low pass and high pass filters H(w) and G(w) respectively.

Let $[h_0, h_1, ..., h_{N-1}]$ and $[g_0, g_1, ..., g_{N-1}]$ be the impulse response of the low pass and high pass filters, respectively. The following conditions ensure orthogonality of transform so that no information is lost in decomposition process and the filter bank is said to enjoy a perfect reconstruction (PR) property [4].

$$\sum_{n=0}^{N-1-2l} h_n h_{n+2l} = \begin{cases} 1 & ,l = 0 \\ 0 & ,0 < l < N/2 \end{cases} \quad (3a)$$

$$g_n = (-1)^{n+1} h_{N-n-1} \quad ,n = 0,1,..., N-1 \quad (3b)$$

2.2 Designing Matched Wavelet

Recently, new methods have been proposed for designing matched wavelets aiming at optimizing the filtering procedure by maximizing the variance explained by the wavelet coefficients or by maximizing the compression performance of the filter banks [4]-[6]. Among the algorithms of this category, the one proposed in [5] leads to better results for the ECG signal classification according to the results presented in [4]. Also the algorithm proposed in [6] has local minima problem since its objective function may have local maxima different from the global maximum [4]. To overcome this problem and to gain more accurate results, the algorithm proposed in [5], is chosen to design the matched wavelets.

Let $\underline{x} = [x_0 x_1 \ ... \ x_{J-1}]$ be the row vector of the original variables. A circulant matrix from the data vector can be formed as :

$$\breve{X} = \begin{bmatrix} x_0 & x_1 & ... & x_{J-2} & x_{J-1} \\ x_1 & x_2 & ... & x_{J-1} & x_0 \\ x_2 & x_3 & ... & x_0 & x_1 \\ \vdots & \vdots & & \vdots & \vdots \\ x_{N-1} & x_N & ... & x_{N-3} & x_{N-2} \end{bmatrix}_{N \times J} \quad (4)$$

where N is the length of the high-pass and low-pass filters impulse response.

Assuming that the filtering is performed by circular convolution, computing the approximation coefficients $\underline{c} = [c_0, ..., c_{J/2-1}]$ and the detail coefficients $\underline{d} = [d_0, ..., d_{J/2-1}]$ is straightforward.

$$\underline{c'} = h\breve{X} \quad (5a)$$

$$\underline{d'} = g\breve{X} \quad (5b)$$

where down sampling $\underline{c'}$ and $\underline{d'}$ generates the approximation and detail coefficients $\underline{c}$, $\underline{d}$, respectively.

The power of the low-pass and high-pass filter outputs can be defined as the energy divided by the number of the coefficients, which for the m^{th} training signal leads to

$$P_c^m = \frac{\underline{c}^m \underline{c}^{m^T}}{J/2} \quad (6a)$$

$$P_d^m = \frac{\underline{d}^m \underline{d}^{m^T}}{J/2} \quad (6b)$$

The superscript *m* identifies the powers, approximations and details of the m^{th} signal. The overall power of the approximations and details for M training signals are

$$P_c = \sum_{m=1}^{M} P_c^m \quad (7a)$$

$$P_d = \sum_{m=1}^{M} P_d^m \quad (7b)$$

An objective function similar to the coding gain can be defined implying the compression performance of two-channel filter bank structures.

$$F(\underline{h}, \underline{g}) = \frac{0.5(P_c + P_d)}{\sqrt{P_c P_d}} \quad (8)$$

When the conditions in (3) are satisfied, the wavelet transform is orthogonal and there is no loss in information. So, the sum of the energy at the outputs of the analysis filters is equal with the power of the original signal, which is a constant value for each training signal. Thus the objective function can be rewritten in following form:

$$F^2 = \frac{0.25A^2}{AP_c - P_c^2} \quad (9)$$

where *A* is the sum of the approximation and detail powers. It is clear that *F* reaches a minimum when $P_c = A/2$ and as it increases from *A/2* to *A*, *F* increases and tends to $+\infty$. Thus, maximizing P_c results in maximizing the objective function *F*. Moreover, the signal to noise ratio (SNR) is usually larger in the low-pass filter output, so maximizing P_c further improves the filtering performance. As a result, a new objective function $\varepsilon: \Re^N \to \Re$, can be defined as follows

$$\varepsilon(h) = P_c = \frac{1}{J/2}\sum_{m=1}^{M} \underline{c}^m \underline{c}^{m^T} \cong \frac{1}{J}\sum_{m=1}^{M} \underline{c}^{'m} \underline{c}^{'m^T} \quad (10)$$

By substituting (5a) in (10), (10) may be rewritten in the following form:

$$\varepsilon(h) = \frac{1}{J}\sum_{m=1}^{M} \underline{h}\breve{X}^m \breve{X}^{m^T} \underline{h}^T = \underline{h}\underbrace{\sum_{m=1}^{M}\left(\frac{1}{J}\breve{X}\breve{X}^T\right)}_{R}\underline{h}^T \quad (11)$$

The R matrix defined in (4) is a Toeplitz matrix since $\breve{X}\breve{X}^T$ is Toeplitz for any data vector. Under the PR (perfect reconstruction) conditions and assuming that $[r_0, r_1, ..., r_{N/2-1}]$ are the elements of the first row of *R*, the objective function can be restated in the following form:

$$\varepsilon(\underline{a}) = \frac{r_0}{2} + \sum_{n=0}^{N/2-1} a_n r_{2n+1} \quad (12)$$

The vector $\underline{a} = [a_0, a_1, ..., a_{N/2-1}]$ contains the coefficients of the product filter $P(z) = H(z)H(z^{-1})$. By enforcing the following restriction on the frequency response of the product filter $Q(f) = P(e^{j2\pi f})$:

$$Q(f) = 1 + 2\sum_{n=0}^{N/2-1} a_n \cos[2\pi f(2n+1)] \geq 0 \quad (13)$$

The transfer function *H (z)* can be recovered from the product filter *P (z)* for a given $\underline{a}$. It would be sufficient to enforce the restriction in (13) only in the interval $0 \leq f \leq 0.5$ due to the periodicity of *Q (f)*. So the optimization problem can be restated as the maximization of $\varepsilon(\underline{a})$ with respect to $\underline{a}$ subject to the restriction defined in (13), which is a linear semi-infinite programming (LSIP) problem. Following the mathematical and computational steps proposed in [5] and applying a conventional optimization technique such as the simplex method, the optimal $\underline{a}$ and as a result, the impulse response of the low-pass filter for the matched wavelet basis is obtained.

For the sake of compatibility with the previous works on wavelet-based ECG feature extraction, MIT-BIH Arrhythmia which is a standard annotated ECG database is selected as the data set both for training and testing the proposed algorithm. The length of all these wavelets is set to N=8, the average of the length of the wavelets used in the related works. The length of the data vectors is 180, which is half of the sampling frequency of the MIT-BIH Arrhythmia database.

Moreover, to follow the recommendations of the AAMI (Association for the Advancement of Medical Instrument) the first 5 minute of each sample is removed. Two datasets, the first one containing 24 records with the even numbers and the second one containing 24 records with the odd numbers from MIT-BIH Arrhythmia database, are selected as the target of matching the designed wavelets; W#1 and W#2, respectively. This approach ensures the random selection of different ECG records. The first 5 minute of these records is eliminated and the first 180-length interval with the R peak located at its centre is assigned to the i^{th} data vector (*i*=1,…,24) for each dataset. The impulse response and the frequency response of the first and second corresponding low-pass filters, h#1 and h#2, are illustrated in Fig. 1and Fig. 2, respectively.

2.3 R Peak Detection Algorithm

The algorithm proposed in [1], [2] is the basis of R peak detection in this work. The algorithm, which is applied directly to the digitized signal without any filtering or pre-processing, contains three main parts: 1) windowing the ECG signals to analyze 720 data points at a time, 2) computing the DWT of each interval at the scales 2^2 and 2^1, and 3) analyzing the wavelet coefficients for R peak detection.

According to signal detection theory, if the system response of the detection filter matches the signal embedded in noise, the signal-to-noise ratio (SNR) is maximized, and a sharp and high peak will be produced at the output showing the maximum correlation [4]. Using the matched wavelets for decomposing the ECG signal, positive maximum-negative minimum pairs are generated at the locations of R peaks in the scales 2^1 and 2^2 (Fig.3). Thus locating these maximum-minimum pairs leads to detection of R peaks in the ECG signal.

Studying the DWT coefficients of the ECG signals when using matched wavelets at different scales and in order to decrease the volume of mathematical computations and processing, it seemed sufficient to analyze the wavelet coefficients only at scales 2^2 and 2^1, since the DWT at these scale seemed to be informative enough for analyzing the signal characteristics.

The R peak detection algorithm can be divided into the following steps:

Step1: The DWT of the signal is calculated at scales 2^2 and 2^1 .

Step 2: All the maxima in DWT larger than a threshold 2.5 times larger than the rms of DWT coefficients at each interval at scale 2^2 are detected.

Step 3: In the neighboring of each maximum-minimum pair detected at scale 2^2 , a similar pair is detected at scale 2^1.

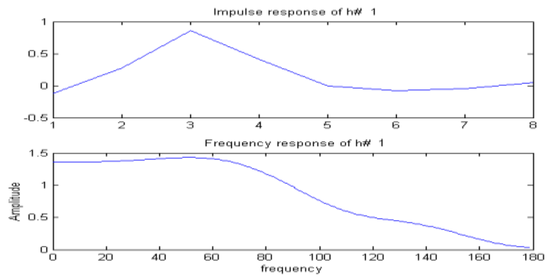

Fig.1. Impulse response and the frequency response of h#1, the low-pass filter of the first designed matched wavelet.

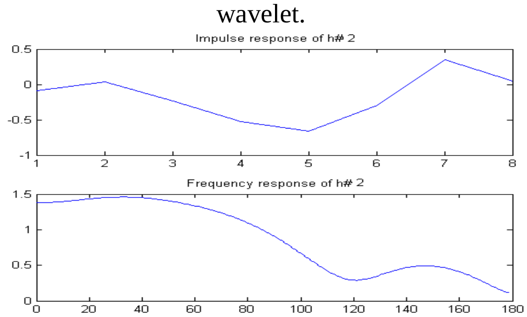

Fig.2. Impulse response and the frequency response of h#2, the low-pass filter of the second designed matched wavelet.

Step 4: Isolated peaks at scale 2^1 are eliminated.

Step 5: Redundant peaks at scale 2^1 are eliminated.

Step 6: The remaining pairs at scale 2^1 correspond to the R peaks of the signal. The zero crossing points of these pairs are calculated and shifted 2 points to the left to obtain the locations of the R peaks in the ECG signal.

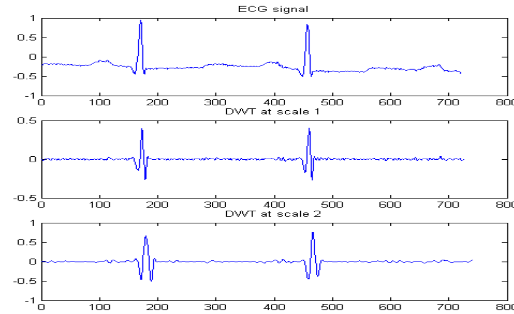

Fig. 3. ECG signal its DWT at scales 2^1 and 2^2. The maximum-minimum pairs at scales 2^1 and 2^2 correspond to R peaks.

3 Experimental Results

The MIT-BIH Arrhythmia database consists of 48 real ECG records whose characteristic points have been carefully annotated. The duration of each record is 30 minutes which has been sampled at 360 Hz. No pre-processing or filtering is applied and the signals are directly analyzed using each of the R peak detection algorithms.

The evaluation process is conducted 3 times. In the experiments 1 and 2, the 2 designed matched wavelets (W#1, W#2) are used for analyzing the ECG records. To compare the efficiency of the designed wavelets with the previously used wavelets, the third experiment is performed using Quadratic Spline wavelet. The wavelet coefficients were analyzed at scales 2^1 and 2^2 in the three experiments.

To evaluate the accuracy of the detection algorithm, the location of the R peaks (which was the output of the algorithm) for each record were compared to the actual location of that record's R peaks, available in the annotations assigned to that specific record. Table I shows the results of the three experiments. "FP" is the number of false positives (incorrectly detected) and "FN" is the number of false negatives (missed) R peaks. The total number of failed detections is the sum of FP and FN.

4 Conclusions

This work presented a new method for wavelet-based R peak detection by using matched wavelets. The matched wavelets were designed with the aim of maximizing the coding gain of the wavelet transform. The previous works required analyzing the wavelet coefficients at 4 different scales while the proposed method needs analyzing the coefficients at only 2 scales. The minimum execution time for the previous methods was about 108 seconds while for the proposed method it was reduced to 26.2 seconds. So, the speed of the algorithm is increased considerably. The results show a considerable increase in the accuracy of the R peak detection algorithm when using matched wavelets instead of Quadratic Spline wavelet, which had led to the best results among the previous wavelet-based R peak detection algorithms.

Table I

THE NUMBER OF FALSE POSITIVE(FP), FALSE NEGATIVE(FN) AND TOTAL FAILED DETECTED R PEAKS USING TWO WAVELET BASES (W#1, W#2) AND QS(QUADRATIC SPLINE WAVELET) FOR MIT-BIH ARRHYTHIMA DATABASE.

Signal	Total beats	FP(QS)	FP(#1)	FP(#2)	FN(QS)	FN (#1)	FN(#2)	FD(#1)	FD(#2)	FD(#3)
100	2273	0	0	0	0	0	0	0	0	0
101	1865	1	4	4	2	3	4	3	7	8
102	2187	0	0	0	3	0	0	3	0	0
103	2084	0	0	0	0	0	0	0	0	0
104	2230	137	38	77	57	35	74	194	73	151
105	2572	21	12	17	26	5	34	47	17	51
106	2027	1	1	19	100	12	49	101	13	68
107	2137	651	18	2	486	21	10	1137	39	12
108	1763	525	171	60	132	173	76	657	344	136
109	2532	3	1	5	184	6	16	187	7	21
111	2124	8	1	5	6	1	16	14	2	21
112	2539	0	0	0	0	0	2	0	0	2
113	1795	0	0	0	0	0	0	0	0	0
114	1879	9	0	7	9	0	18	18	0	25
115	1953	0	0	0	0	0	0	0	0	0
116	2412	13	0	6	48	0	2	61	0	8
117	1535	2	0	0	1	0	1	3	0	1
118	2275	4	0	4	3	0	5	7	0	9
119	1987	0	0	0	445	0	0	445	0	0
121	1863	2	0	1	2	1	37	4	1	38
122	2476	0	0	0	0	0	0	0	0	0
123	1518	0	0	0	2	2	3	2	2	3
124	1619	33	0	3	4	0	16	37	0	19
200	2601	118	9	31	64	5	8	182	14	39
201	1963	9	47	26	66	14	15	75	61	41
202	2136	0	0	0	162	304	311	162	304	311
203	2982	124	272	105	119	199	332	243	471	437
205	2656	0	0	0	33	16	18	33	16	18
207	1862	317	272	191	90	114	38	407	386	229
208	2956	19	37	32	642	49	36	661	86	68
209	3004	6	0	0	18	0	0	24	0	0
210	2647	14	4	23	77	66	26	91	70	49
212	2748	2	0	4	3	0	1	5	0	5
213	3251	1	0	4	175	8	3	176	8	7
214	2208	8	8	22	26	18	18	34	26	40
215	2154	9	4	5	16	2	4	25	6	9
217	2208	14	2	3	42	5	8	56	7	11
219	2154	6	0	0	26	0	0	32	0	0
220	2048	0	0	0	30	10	14	30	10	14
221	2427	1	7	7	24	23	11	25	30	18
222	2484	16	9	46	58	26	57	74	35	103
223	2605	0	0	2	11	4	46	11	4	48
228	2053	168	56	112	21	28	104	189	84	216
230	2256	0	0	0	0	1	0	0	1	0
231	1886	0	0	0	1	0	0	1	0	0
232	1780	588	284	140	5	13	14	593	297	154
233	3079	0	2	0	71	2	8	71	4	8
234	2753	0	0	1	2	0	0	2	0	1
Total	**116137**	**2830**	**1259**	**964**	**3292**	**1166**	**1435**	**6122**	**2425**	**2399**

Future work will use further processing of the wavelet coefficients in order to increase the efficiency and reduce the effect of noise in the ECG signals.

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