EXACT RECOVERY LIMITATIONS OF MEG SOURCE LOCALIZATION MODEL FOR EPILEPSY

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ABSTRACT

Source localization in magnetoencephalography (MEG) has received a lot of attention in the recent years. MEG inverse problem is an ill-posed problem in from of $\boldsymbol{B}_{n\times 1}=\boldsymbol{G}_{n\times m}\boldsymbol{J}_{m\times 1}$, where $\boldsymbol{B}$ is the vector of magnetic fields measured by the magnetic sensors, $\boldsymbol{G}$ is the lead field matrix, and $\boldsymbol{J}$ is the vector of source current in the brain. In practice, $n<<m$ and $\boldsymbol{J}$ is usually a sparse vector. In this paper, we employ a complete reconstruction criterion to estimate the maximum number of active sources in the model or equivalently the volume of the active areas that guarantees a stable solution to the MEG inverse problem. Based on some prior information about the volume of the active areas, we then evaluate the stability of the solution with the employed model. Our approach guarantees an error upper band on the solution with certain number of active points. In the experiments, we have demonstrated the technique using some simulations.

1. INTRODUCTION

Localizing the brain neural activities is of special interest for research and diagnosis applications. Over the last decade, functional magnetic resonance imaging (fMRI) and magnetoencephalography (MEG) have received increased attention to this aim. fMRI measures the brain activities with poor temporal resolution (in the order of seconds) but good spatial resolution (in the order of millimeter). In contrast, MEG provides a good temporal resolution (in the order of milliseconds) but poor spatial resolution. In MEG, the neural activities of the brain are generally modeled by current dipoles [1, 2]. Based on prior knowledge about the brain anatomy and function, the locations and orientations of the dipoles are predetermined. However, the amplitude of each dipole current is unknown. Having the magnitude of the magnetic field at some points outside the brain, this problem is transformed into the inverse problem of:

$$\boldsymbol{B}_{n\times 1}=\boldsymbol{G}_{n\times m}\boldsymbol{J}_{m\times 1} \tag{1}$$

where $\boldsymbol{B}$ is the vector of magnetic fields measured by the magnetic sensors, $\boldsymbol{G}$ is the lead field matrix, and $\boldsymbol{J}$ is the vector of source current in the brain [2]. The goal is to estimate the source current $\boldsymbol{J}$ from the observed magnetic field $\boldsymbol{B}$. In practice, $n<<m$, i.e., the number of measurements is far lower than the number of dipoles. Nevertheless, $\boldsymbol{J}$ is usually a sparse vector. For example, in the case of localizing the seizure activities in epileptic patients, the number of active sources is limited; resulting in a sparse $\boldsymbol{J}$.

Conventionally, the above inverse problem is solved by minimizing the following cost function:

$$E(\boldsymbol{J})=\|\boldsymbol{B}-\boldsymbol{G}\cdot\boldsymbol{J}\|^2+\lambda\cdot\boldsymbol{J}'\cdot\Sigma_0\cdot\boldsymbol{J} \tag{2}$$

The first term in the above equation is the reconstruction error, which measures the difference between the observed MEG data and the MEG signal produced by the estimated dipoles. To resolve the ill-posed nature of the inverse problem and incorporate the noise effect, a regularization term (the second term in equation (2)) is added to the cost function. Prior information about the problem may be incorporated in Σ_0. A stable recovery of the signal $\boldsymbol{J}$ is important to have a reliable localization of the brain activities. By stable recovery we mean that small changes in the observations should result in small changes in the recovered signal.

Many practical solutions are studied in the resent years, namely FOCUSS [4] and LORELTA [5]. Although most of proposed algorithms guarantee convergence under certain circumstances, in the worse cases they converge to unexpected solutions. This observation raises this question that given a problem, how much accuracy we should expect.

It is difficult to determine the appropriate number of active dipole sources in the MEG inverse problem. Conventionally, some dipoles are considered inside the brain and the inverse problem is solved by optimization techniques. The solution determines what dipoles are active, which specifies the active areas of the brain. Candes et al. [3] have proposed a procedure to determine whether or not a stable solution to this inverse problem is guaranteed. In this paper, we employ the proposed method in [3] for the MEG inverse problem. We do not provide a solution to the MEG inverse problem; rather we evaluate the stability of the solution. The proposed method is tested by conducting some simulations.

The outline of this paper is as follows. In Section 2, we introduce the proposed evaluation technique in [3] for a stable recovery. In Section 3, we provide the simulations' and their results. Finally we conclude and discuss in Section 4.

2. METHOD

2.1. Sparse Solution Stable Recovery

The MEG inverse problem from imperfect observations can be described as follows:

Given a limited number of contaminated observations $y = Ax_0 + e$, where A is an n×m matrix, y and x_0 are respectively n×1 and m×1 vectors, and e is an error term, recover the signal $x_0 \in \Re^m$ whose support $T_0 = \{t : x_0(t) \neq 0\}$ is assumed to have a small cardinality. A usually has far fewer rows than columns ($n \ll m$). This means that the number of observations is far lower than the number of unknown values. To recover x_0, we may consider the solution $x^{\#}$ to the l_2-regularization problem

$$\min \|x\|_{l_2} \quad \text{subject to} \quad \|Ax - y\|_{l_2} \leq \varepsilon \tag{3}$$

where ε is the size of the error term e. To introduce the concept of stable recovery, let A_T, $T \subset \{1, \ldots, n\}$ be the $n \times |T|$ submatrix obtained by extracting the columns of A corresponding to the indices in T. Then, define the S-restricted isometry constant δ_S of A which is the smallest quantity such that

$$(1 - \delta_S)\|c\|_{l_2}^2 \leq \|A_T c\|_{l_2}^2 \leq (1 + \delta_S)\|c\|_{l_2}^2 \tag{4}$$

for all subsets T with $|T| \leq S$ and coefficient sequences c_j, $j \in T$. The following theorem provides a sufficient condition to guarantee a stable recovery [3]:

Theorem:
Let S be such that $\delta_{3S} + 3\delta_{4S} < 2$. Then for any signal x_0 supported on T_0 with $|T_0| \leq S$ and any perturbation e with $\|e\|_{l_2} \leq \varepsilon$, the solution $x^{\#}$ to (3) obeys

$$\|x^{\#} - x_0\|_{l_2} \leq C_S \cdot \varepsilon \tag{5}$$

where the constant C_S may only depend on δ_{4S}. For reasonable values of δ_{4S}, C_S is well behaved; e.g. $C_S \approx 8.82$ for $\delta_{4S} = 1/5$ and $C_S \approx 10.47$ for $\delta_{4S} = 1/4$.

To use the above theorem for MEG inverse problem, we just need to replace $B = y$, $G = A\Sigma_0^{0.5}$, and $J = \Sigma_0^{-0.5}x$ in the above theorem. We can rewrite Equation 3 as:

$$\min J' \cdot \Sigma_0 \cdot J \quad \text{subject to} \quad \|G \cdot J - B\|_{l_2} \leq \varepsilon \tag{6}$$

Using Lagrange coefficients theory, the optimization problem in Equation 2 obtains:

$$E(\boldsymbol{J}) = \boldsymbol{J}' \cdot \Sigma_0 \cdot \boldsymbol{J} + \frac{1}{\lambda} \cdot \|\boldsymbol{B} - \boldsymbol{G} \cdot \boldsymbol{J}\|^2 \tag{7}$$

Then, S is the maximum number of active dipoles in the MEG model.

Assuming c is not ***0***, Equation 4 could rewrite as

$$\begin{cases} (1 - \delta_S) \leq \|G_T' \tilde{c}\|_{l_2}^2 \leq (1 + \delta_S) \\ s.t.\ \|\tilde{c}\|_{l_2} = 1 \end{cases} \tag{8}$$

Where $G' = G \cdot \Sigma_0^{-0.5}$ That implies

$$\max_{|T| \leq S} [\lambda_{\max}^2 (G_T') - 1, 1 - \lambda_{\min}^2 (G_T')] \leq \delta_S \tag{9}$$

where $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ are representing maximum and minimum eigenvalue functions, respectively.

2.2. Stable MEG Localization

As stated before, in a typical MEG problem, the location and orientation of the dipoles in the model, as well as the location of the sensors are known from some prior information. This means the matrix A is already known. On the other hand, based on (4), δ_S is purely dependent on matrix A. Therefore, knowing A we may calculate δ_S for S=1, 2, … and determine the maximum value of S that satisfies the condition $\delta_{3S} + 3\delta_{4S} < 2$. This value of S is the maximum number of active dipoles that guarantee a stable signal recovery. This may be used to evaluate the model of dipoles as follows. The number of active dipoles S or equivalently the volume of the active areas is first estimated using some prior information [4]. Now if this value does not satisfy the condition $\delta_{3S} + 3\delta_{4S} < 2$, the solution is not guaranteed to be stable. In that case, we may re-define the model by decreasing the resolution and lowering the number of dipoles in the MEG model. This may be repeated until a stable recovery is guaranteed.

Smart choice of the resolution in each step could guarantee an error band on the location of results.

In conclusion, this construction is suggesting a low-resolution to high-resolution approach to guarantee an error bound. In contrast to traditional FOCUSS algorithm, the resolution of answers in each step is determining using stable recovery factor in (9).

3. SIMULATIONS

3.1. Simulation 1: Simple Sphere Model

In this experiment, the brain is modeled with a sphere and n sensors are considered around the brain and m dipoles are distributes uniformly inside the brain as potential sources. In these simulations very simple homogeny model is used to calculate the G matrix.

Figure 1 shows the stable recovery criteria for 30 and 60 dipoles. As we can see, in identical geometry, we the resolution increases form 30 pixels to 60 pixels, number of stable recoverable point decrease form 5 to 3. In the second simulation the number of dipoles is increased from 800 to 2000 and stable recoverable point decrease form 60 to 35. As we can as resolution is increased, smaller activity is accurately detectable. The result is hold when we have generally small real activity area compare to number of dipoles. This condition is necessary to keep stable recovery condition valid.

3.2. Simulation 2: MRI Based Model

In this experiment, we used a realistic MRI based model for the brain and considered 120 sensors around the brain. The G matrix is calculated using MEG-Tools [6] forward modeling algorithm base on MRI image of 35 years old patient with temporal lobe epilepsy.

The MEG signal is simulated using the forward modeling. Also measurement noise is considered to simulate artifacts. To recover the signal location, we use two models: the actual forward model that previously used for signal generation and simplified homogenous model that just use the boundary of MRI images. In both simulations we exact co-registration between MRI and MEG.

Figure 2 shows the average localization error verses number of active points. As we can see, FOCUSS error increases dramatically when we have more than 50 active points. Also comparing result of two simulations, we can observe that complex forward model is more sensitive to high number of active points. Although in the small number of active points, complex model has a significant benefit, the gap between performances of two models reduce as number of active points increases.

By selecting the resolution with respect to forward model and number of active points, localization error dramatically reduces in larger actvities.

The point is when the number of active points is low, the initial resolution used by FOCUSS algorithm is less than stable recovery threshold; however, by increasing the activation number violate the stable recovery condition.

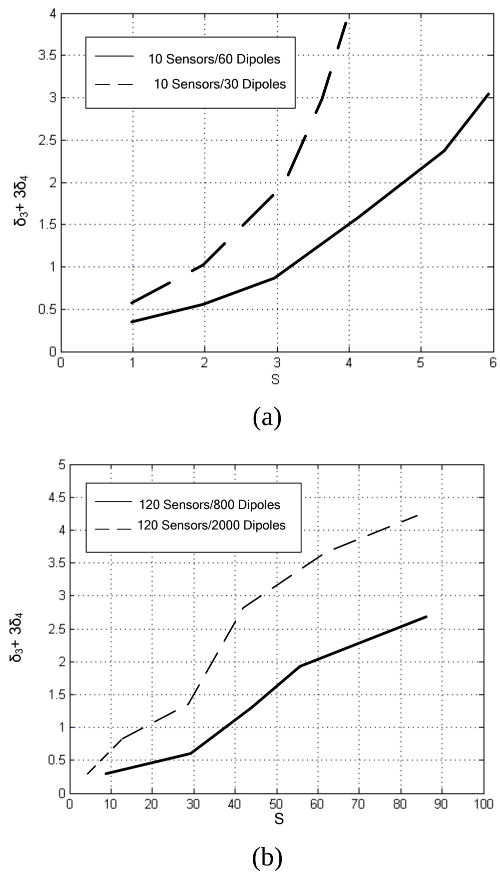
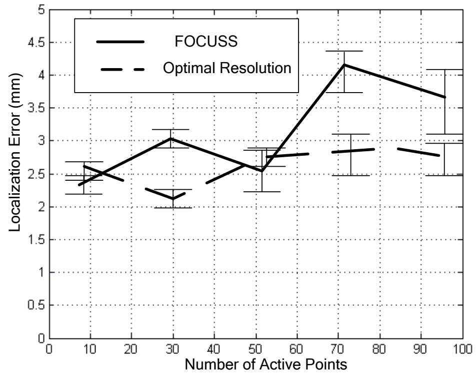


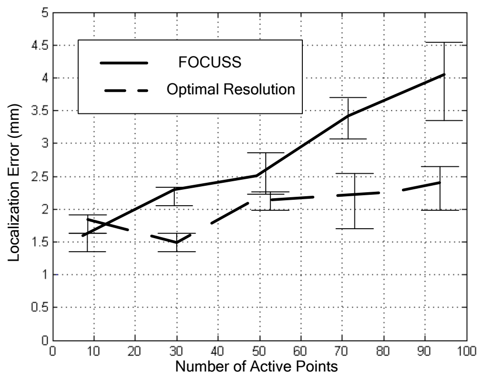
Fig. 1. Effect of number of dipoles on the stable recovery threshold: in identical geometry, when number of dipoles increase, number of recoverable point decrease. a. simulation using 10 sensors, b. simulation using 120 sensors.

4. CONCLUSION AND DISCUSSION

Using stable recovery criterion show how the accuracy of the result depends on prescience of activity area size complexity of model.



(a)



(b)

Fig. 2. Localization error verses number of active points. a. Simplified forward model and b. real complex forward model.

5. REFERENCES

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